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TECHNICAL MEMORANDUM 115/58-PREP

A STUDY OF A SIMPLIFIED THEORY OF GRAVITY CONCENTRATION IN COAL JIGS*

by

R. P. Charbonnier

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ABSTRACT

To understand the operation of jigs and other separating processes based on differences in gravity, it is desirable to analyse the motion of various particles in given media under specific conditions.

A summary of the earlier theories shows that they lead to unsatisfactory or excessively complicated formulae in cases such as coal jigging.

By adopting a simplified theory of the motion of particles in a fluid, the size and duration of the initial accelerated motion of a particle falling in hindered settling can be calculated. Based on this mathematical analysis, examples referring to actual coal jigging practice show that it is desirable to prolong and/or repeat, as frequently as possible, the optimum conditions corresponding to an effective acceleration of the particles. The separation is then obtained according to gravity rather than to particle dimensions, as would be the case if the velocity of the particles were allowed to approach its terminal value. To achieve this gravity separation, it is suggested that the suction phase of the jigging cycle be artificially prolonged and the frequency of pulsations increased as much as possible for washing the coals of smaller sizes.

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INTRODUCTION

In order to obtain a proper understanding of the operation of jigs or other processes of mineral concentration using gravity separation, it is advisable to analyse the motions, in various media, of particles of different specific gravities and dimensions. The study of the movement of solids in fluids is of utmost importance, not only in mineral dressing processes such as classification, thickening, filtration and gravity concentration, but, more generally, in the vaster fields of hydrodynamics and aerodynamics. This paper, however, deals only with the motion of coal or rock particles in water or media of similar density and viscosity, under the conditions usually obtained in coal jigs. These conditions are characterized by low fluid velocities, intermediate temperatures and viscosities, and low proportions of very fine particles, thus generally corresponding to relatively low values of Reynolds' number.

EARLIER THEORIES

One of the main difficulties in such studies is the evaluation of the forces of resistance opposing the movement of the particle, including the force resulting from interaction with neighbouring particles under hindered settling conditions.

We are here concerned primarily with the theory of jigging, and, more particularly, of coal jigging. In this case, the earlier theories are valid only within very limited ranges of conditions, usually different from those found in practice. Stokes' law applies only to the relatively rare case of laminar or viscous flow for a very small isolated sphere (radius (r) between 0.1 and 50 microns) moving at slow speed (v) in an infinite medium of viscosity (b). Then, Stokes' law (1) states that the resistance is:

$$R = 6\pi brv \text{ in c.g.s. units}$$

The density of the fluid, which does not appear in Stokes' law, had been introduced a century earlier in Newton's law (2), which stated that the resistance to the motion of a coarse sphere of radius (r) and

velocity (v) in a medium of density (u) is:

$$R = \frac{\pi}{2} u r^2 v^2$$

However, Newton's relationship is not satisfactorily verified by experimental evidence. Rittinger's equation (3) is a modification of Newton's law by the introduction of a "coefficient of resistance" (Q), giving the resistance as:

$$R = Q \frac{\pi}{2} u r^2 v^2$$

The Newton-Rittinger relation may be acceptable only for a relatively large isolated sphere (of radius exceeding 0.2 cm) at moderate velocities in a turbulent or eddying flow. Then, the coefficient Q may be considered as constant (approximately 0.4 in c.g.s. units).

These conditions are not often found in usual jigging practice; they correspond to Reynolds' numbers ranging from 1,000 to 200,000.

In the case of a sphere, the Reynolds' number is:

$$R.N. = \frac{2vru}{b}$$

Stokes' law is acceptable for a Reynolds' number of one (R.N. = 1)or less. Conditions with Reynolds' numbers exceeding 200,000, which have been studied by Eiffel and Prandtl, are not found in coal jigging.

In the case of conditions intermediary between the ranges of application of Stokes' law and Newton-Rittinger's relation, the general Allen equation (4) for the free settling of a sphere in a gaseous fluid is:

$$R = K b^{2-n} . u^{n-1} . r^n . v^n$$

with the same notations as above and where K and n are constants dependent upon the velocity (v). For the intermediate conditions, Allen's equation for $n = 3/2$ resolves to:

$$R = K b^{1/2} u^{1/2} r^{3/2} v^{3/2}$$

Thanks to the use of the fractional exponents 1/2 and 3/2, which correspond to values intermediary between those used in Stokes' and Rittinger's laws, the Allen equation is somewhat more adequate for the transition range of conditions encountered in jigging practice. However, this equation still applies only in particular cases, not too frequently found in coal jigging.

The Castleman function (5) deals only with the terminal velocity of free settling spheres, although it covers graphically a very wide range of conditions. Many formulae have been proposed, by Oseen, Gessner, Goldstein, Budryk, Wadell and others, to be applied in the transition range of conditions, but none of them can be wholly suitable in the relatively wide range of conditions found in coal jigging.

The various influences of the shape of the particle, of the so-called wall effect, and of the simultaneous presence of several particles (hindered settling), have been investigated by many workers, and correction factors have been proposed, notably by Ladenburg-Faxen, Francis, Munroe, Einstein, Guth, Gold, Kermack, McKendrick, Ponder and, especially, Richards (6) and Wadell. Each of these studies has dealt more specifically with a limited range of conditions, and the main variable under consideration has generally been the terminal velocity.

A SIMPLIFIED THEORY OF JIGGING

During extensive experiments on a Vissac jig of a new automatic feldspar type, at the newly rebuilt coal preparation plant of Nordegg, Alberta, where he was Mining Engineer, the author has had the privilege of working with Mr. G.A. Vissac (7) on a simplified equation of motion, which is based on data obtained in practical coal jigging and which permits a mathematical analysis of the movement of a particle in a jig. The resistance to the hindered motion of a particle falling under the conditions usually encountered in jigging was assumed to be of the Newton-Rittinger type: $R = a u v^2$,

with the same notations as previously, and where (a) is the cross-section area of the particle, falling in a medium of apparent density (u).

For most practical purposes, the Vissac formulae (7) for the resistance to motion are satisfactorily in agreement with experimental data (such as Richards') within the usual ranges of conditions found in coal jigging, and yet they are relatively simple and may therefore be used for mathematical calculations of several important variables in addition to the terminal velocity. These calculations will be given here since the methods used in this case may be applied to the study of similar problems.

As a first approximation, the medium of solid particles in suspension is assimilated to a homogeneous fluid having an apparent specific gravity calculated according to its composition. This practical assumption is now widely accepted (8) and is satisfactorily confirmed by experimental data under hindered settling conditions.

The mathematical expression of the resistance, which consists of the tangential "skin friction drag" and the "pressure or form drag" (9), is simplified in accordance with the results of classic experiments in fluid dynamics (10), and the resistance of the fluid to the movement of a particle is given as twice the value of the frontal resistance, which is equal to the kinetic energy of the unit volume of displaced fluid. These approximations can be accepted under usual conditions of coal jigging, inasmuch as the final results of the calculations are in satisfactory agreement with experimental data such as given by Richards and DeCaux.

In the case of a particle of density (s), thickness (L) and cross-section area (a), falling in hindered motion in a medium of apparent density (u), it is submitted, when its velocity is (v), to the following

forces: a constant force (W) equal to the gravity less the buoyant forces, with

$$W = L\,a\,(s - u)\,g\,,$$

where (g) is the acceleration of gravity; and the resistance (R) of the fluid to the motion, with

$$R = a\,u\,v^2\,.$$

The differential equation of motion of the particle falling with the acceleration $(\frac{dv}{dt})$ in a motionless medium is, therefore:

$$L\,a\,s\,\frac{dv}{dt} = L\,a\,(s - u)g - a\,u\,v^2$$

or:

$$\frac{dv}{dt} = \frac{g}{L\,s}\cdot\left[L\,(s - u) - \frac{u}{g}\,v^2\right] \qquad (1)$$

The initial acceleration is obtained for $v = 0$ and is:

$$A_o = \left(\frac{dv}{dt}\right)_o = g\,(1 - \frac{u}{s}) \qquad (2)$$

A_o depends only on the specific gravity of the particle and not on its dimensions. The initial accelerated motion thus permits efficient separation, but it is soon followed by an almost uniform motion with a velocity close to the terminal velocity (V_t), which is obtained for $\frac{dv}{dt} = 0$ and is:

$$V_t = \sqrt{g\,L\,(\frac{s - u}{u})} \qquad (3)$$

Therefore, (V_t) depends on both the specific gravity and the dimensions of the particle. The optimum conditions for gravity separation are thus obtained during the initial accelerated fall. These conditions are repeated as frequently as possible in jigs by pulsating motions of the medium.

Consequently, it is very important to calculate the size and duration of the initial accelerated motion.

ANALYSIS OF THE MOTION

Duration of the Accelerated Motion

The equation of motion (1) can be written:

$$\frac{dv}{dt} = g\left(\frac{s-u}{s}\right)\left[1 - v^2 \frac{u}{g\,L\,(s-u)}\right]$$

and becomes, by substitution of equations (2) and (3),

$$\frac{dv}{dt} = A_o(1 - v^2/V_t^2) \tag{4}$$

$$t = \frac{1}{A_o}\int \frac{dv}{1 - v^2/V_t^2}$$

$$t = \frac{1}{A_o}\,(-V_t/2)\,\mathrm{Log}_e\left(\frac{1 - v/V_t}{1 + v/V_t}\right) + C$$

If $t = 0$, we have $v = 0$, and therefore the integration constant $C = 0$

$$t = -\frac{V_t}{2A_o}\,\mathrm{Log}_e\left(\frac{1 - v/V_t}{1 + v/V_t}\right) \tag{5}$$

The duration (T) of the accelerated motion is infinite since it is obtained for $v = V_t$. However, an approximate value of (T) can be obtained when the velocity of the particle is v_1 equal to 90% of the terminal velocity V_t. Then the acceleration is:

$$A_1 = \left(\frac{dv}{dt}\right)1 = A_o\,(1 - v_1^2/V_t^2)$$

As $v_1 = 0.9\,V_t$, we have $A_1 = 0.19\,A_o$

If this acceleration A_1 is not regarded as negligible, another percentage of V_t can be used for the calculation such as $v_2 = 0.95\,V_t$.

The duration (t) of the accelerated motion of the particle up to the time when its velocity is 90% of the terminal velocity will be obtained by replacing (v) by (v_1) in equation (5). If desirable, some other percentage of the terminal velocity may be used.

Distance of Fall in Accelerated Motion

From equation (4): $\frac{dv}{dt} = A_o(1 - v^2/V_t^2)$ (4)

we have: $A_o\, dt = dv/(1 - v^2/V_t^2)$

Hence:

$$v = V_t(1 - e^{-2(A_o/V_t)t}) / (1 + e^{-2(A_o/V_t)t}) + C \qquad (6)$$

If $t = 0$, we have $v = 0$ and therefore the integration constant $C = 0$.

Then, if (h) is the distance of fall of the particle, we have: $dh/dt = v$ and by integration:

$$h/V_t = \int dt - 2\int \frac{e^{-2(A_o/V_t)t}}{1 + e^{-2(A_o/V_t)t}}\, dt$$

$$= t + (V_t/A_o)\, \mathrm{Log}_e\, (1 + e^{-2(A_o/V_t)t}) + C$$

If $t = 0$, we have $h = 0$ and the integration constant $C = -(V_t/A_o)\, \mathrm{Log}_e\, 2$.

Hence:

$$h = V_t\, t + (V_t^2/A_o)\, \mathrm{Log}_e\, \left(\frac{1 + e^{-2(A_o/V_t)t}}{2}\right) \qquad (7)$$

From equations (2) and (3), we have: $V_t^2/A_o = Ls/u$

From equation (5), we have:

$$e^{-2(A_o/V_t)t} = (1 - v/V_t) / (1 + v/V_t)$$

By substituting this value in (7), and also replacing (t) by formula (5) and V_t by formula (3), we obtain:

$$h = -\frac{Ls}{2u}\left[\mathrm{Log}_e\, \left(\frac{1 - v/V_t}{1 + v/V_t}\right) + 2\, \mathrm{Log}_e\, (1 + v/V_t)\right]$$

$$h = -\frac{Ls}{2u}\, \mathrm{Log}_e\, (1 - v^2/V_t^2) \qquad (8)$$

If the expressions (6) and (7) of the velocity (v) and the distance (h) in function of time are not desired, the relation (8) can be obtained much more quickly as follows:

From equation (4) we have:

$$v = \frac{dh}{dt} = \frac{dh}{dv} \cdot \frac{dv}{dt} = \frac{dh}{dv} A_o (1 - v^2/V_t^2)$$

Hence: $$h = \int dh = \frac{1}{A_o} \int \frac{vdv}{1 - v^2/V_t^2} = \frac{-V_t^2}{2A_o} \text{Log}_e (1 - v^2/V_t^2) + C$$

If $t = 0$, we must have $h = 0$ and therefore the integration constant $C = 0$.

From equations (2) and (3), we have:

$$V_t^2/A_o = Ls/u$$

and thus this expression of (h) is the same as equation (8).

The distance of fall (H) of the particle in accelerated motion is infinite since it is obtained for $v = V_t$.

However, here again, an approximate value of the fall occurring while the acceleration is reasonably efficient can be obtained up to the point where the velocity of the particle is $v_1 = 90\%$ of the terminal velocity V_t, or a higher percentage if desired.

EXAMPLES

(1) $L = 10$ cm, $s = 2$, $v_1 = 0.9\ V_t$

With $u = 1$, we find: $t = 3/10$ second and $h = 16.6$ cm.

With $u = 1.5$, we find: $t = 3.5/10$ second and $h = 11.1$ cm.

(2) $L = 1$ mm, $s = 2$, $v_1 = 0.9\ V_t$

With $u = 1$, we find: $t = 3/100$ second and $h = 1.7$ mm.

With $u = 1.5$, we find: $t = 3.5/100$ second and $h = 1.1$ mm.

CONCLUSIONS

Even though the assumptions on which this mathematical analysis is based may not be perfectly correct, the results obtained from the calculated formulae for the terminal velocity and the duration and distance of fall in accelerated motion can be favourably compared with experimental data, and they definitely show that the motion completed with notable acceleration is of very short duration and extremely limited amplitude for very small particles. Therefore, unless the frequency of pulsations is rather high, each particle under consideration will be submitted, during most of the time, to a motion with uniform velocity approximately equal to the asymptotic terminal velocity, instead of to the initial accelerated motion which corresponds to the optimum conditions for separation according to gravity.

Instead of repeating more frequently these optimum conditions by increasing the frequency of pulsations, which may be limited for mechanical reasons, these conditions may be prolonged by extending the suction phase over a longer period, thereby maintaining within a low range the relative velocity of the particles in the accelerating descending medium; in this way the initial high acceleration motion can be obtained during a longer part of the jigging cycle. Attempts to produce a somewhat longer suction phase have been quite successful in many cases of fine-coal jigging, but it seems that better gravity concentration could be possible in jigs washing fine coal if the duration of the suction phase were artificially prolonged and, at the same time, the frequency of pulsations were increased by reducing to the utmost the duration of the upward stroke or pulsion phase.

This combined operation may be facilitated by using a battery of small, well controlled jigs, instead of one large jig containing a great mass of water in rather uncontrolled oscillation.

ACKNOWLEDGMENT

Acknowledgment is hereby made of Mr. G. A. Vissac's work on the theory of jigging and of Dr. J. Visman's contribution to this paper.

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ABSTRACT

To understand the operation of jigs and other separating processes based on differences in gravity, it is desirable to analyse the motion of various particles in given media under specific conditions.

A summary of the earlier theories shows that they lead to unsatisfactory or excessively complicated formulae in cases such as coal jigging.

By adopting a simplified theory of the motion of particles in a fluid, the size and duration of the initial accelerated motion of a particle falling in hindered settling can be calculated. Based on this mathematical analysis, examples, referring to actual coal jigging practice, show that it is desirable to prolong and/or repeat, as frequently as possible, the optimum conditions corresponding to an effective acceleration of the particles. Then, the separation is obtained according to gravity rather than to particle dimensions, as would be the case if the velocity of the particles were allowed to approach its terminal value. For this purpose, it is suggested that the suction phase of the jigging cycle be artificially prolonged and the frequency of pulsations increased as much as possible for washing the coals of smaller sizes.

INTRODUCTION

In order to obtain a proper understanding of the operation of jigs or other processes of mineral concentration using gravity separation, it is advisable to analyze the motions of particles of different specific gravities and dimensions in various media. Indeed, the study of the movement of solids in fluids is of utmost importance not only in mineral dressing processes such as classification, thickening, filtration and gravity concentration, but more generally in the vaster fields of hydrodynamics and aerodynamics.

EARLIER THEORIES

One of the main difficulties in such studies is the evaluation of the forces of resistance opposing the movement of the particle, including the force resulting from interaction with neighbouring particles, under hindered settling conditions.

We are here concerned primarily with the theory of jigging, and more particularly coal jigging. In this case, the earlier theories are valid only within very limited ranges of conditions, usually different from those found in practice. Stokes' law applied only to the relatively rare case of laminar or viscous flow for a very small isolated sphere (radius (r) between 0.1 and 50 microns) moving at slow speed (v) in an infinite medium of viscosity (b). Then, Stokes' law (1) states that the resistance is:

$$R = 6\pi brv \text{ in c.g.s. units}$$

The density of the fluid, which does not appear in Stokes' law, had been introduced a century earlier in Newton's law (2), stating that the resistance to the motion of a coarse sphere of radius (r) and

velocity (v) in a medium of density (u) is:

$$R = \frac{\pi}{2} ur^2v^2$$

however, Newton's relationship is not satisfactorily verified by experimental evidence. Rittinger's equation (8) is a modification of Newton's law by the introduction of a "Coefficient of resistance" (Q), giving the resistance as:

$$R = Q \frac{\pi}{2} ur^2v^2$$

The Newton-Rittinger relation may be acceptable only for a relatively large isolated sphere (of radius exceeding 0,2 cm) at moderate velocities in a turbulent or eddying flow. Then, the coefficient Q may be considered as constant (approximately 0,4 in c.g.s. units).

These conditions are not often found in usual jigging practice; they correspond to Reynolds' numbers ranging from 1,000 to 200,000.

In the case of a sphere, the Reynolds' number is:

$$R.N. = \frac{2vru}{b}$$

Stokes' law is acceptable for a Reynolds' number of one (R.N. = 1) or less. Conditions with Reynolds' numbers exceeding 200,000 which have been studied by Eiffel and Prandtl are not found in coal jigging.

In the case of conditions intermediary between the ranges of application of Stokes' law and Newton-Rittinger's relation, the general Allen equation (4) for the free settling of a sphere in a gaseous fluid is:

$$R = Kb^{2-n} . u^{n-1} . r^n . v^n$$

with the same notations as above and where K and n are constants dependent upon the velocity (v). For the intermediate conditions, Allen's equation for $n = 3/2$ resolves to:

$$R = Kb^{1/2} u^{1/2} r^{3/2} v^{3/2}$$

Thanks to the use of the fractional exponents $^1/_2$ and $^3/_2$, which correspond to values intermediary between those used in Stokes' and Rittinger's laws, the Allen equation is somewhat more adequate for the transition range of conditions encountered in jigging practice. However, this equation still applies only in very particular cases, not too frequently found in coal jigging.

The Castleman function (5) deals only with the terminal velocity of free settling spheres, although it covers graphically a very wide range of conditions. Many formulas have been proposed by Oseen, Gessner, Goldstein, Budryk, Wadell and others, to be applied in the transition range of conditions, but none of them can be suitable in the relatively wide range of conditions found in coal jigging.

The various influences of the shape of the particle, of the so-called wall effect and of the simultaneous presence of several particles (hindered settling) have been investigated by many workers, and correction factors have been proposed, notably by Ladenburg-Faxen, Francis, Munroe, Einstein, Guth, Gold, Kermack, McKendrick, Ponder and specially Richards (6) and Wadell. Each of these studies has dealt more specifically with a limited range of conditions and the main variable under consideration has generally been the terminal velocity.

SIMPLIFIED THEORY OF JIGGING

The author has had the privilege of working with Mr. G. A. Visaac (7) on a simplified equation of motion, which permits a mathematical analysis of the movement of a particle in a jig. The resistance to the hindered motion of a particle falling under the conditions usually encountered in jigging was assumed to be of the Newton-Rittinger type:

$$R = a u v^2$$

with the same notations as previously, and where (a) is the cross-section area of the particle, falling in a medium of apparent density (u).

For most practical purposes, the Vissac formulae (7) for the resistance to motion are satisfactorily in agreement with experimental data (such as Richards') within the usual ranges of conditions found in coal jigging, and yet they are relatively simple and may therefore be used for mathematical calculations of several important variables in addition to the terminal velocity. These calculations will be given here since the methods used in this case may be applied to the study of similar problems.

As a first approximation, the medium of solid particles in suspension is assimilated to a homogeneous fluid, having an apparent specific gravity calculated according to its composition. This practical assumption is now widely accepted (8) and is satisfactorily confirmed by experimental data under hindered settling conditions.

The mathematical expression of the resistance, consisting of the tangential "skin friction drag" and the "pressure or form drag" (9) is simplified in accordance with the results of classic experiments in fluid dynamics (10), and the resistance of the fluid to the movement of a particle is given as twice the value of the frontal resistance, which is equal to the kinetic energy of the unit volume of displaced fluid. These approximations can be accepted under usual conditions of coal jigging, inasmuch as the final results of the calculations are in satisfactory agreement with experimental data such as given by Richards and DeCaux.

In the case of a particle of density (s), thickness (L) and cross-section area (a), falling in hindered motion in a medium of apparent density (u), it is submitted, when its velocity is (v), to the following

forces: a constant force (W) equal to the gravity less the buoyant forces, with

$$W = L\,a\,(s - u)\,g$$

where (g) is the acceleration of gravity; and the resistance (R) of the fluid to the motion, with

$$R = a\,u\,v^2$$

The differential equation of motion of the particle falling with the acceleration $(\frac{dv}{dt})$ in a motionless medium is therefore:

$$L\,a\,s\,\frac{dv}{dt} = L\,a\,(s - u)g - a\,u\,v^2$$

of:

$$\frac{dv}{dt} = \frac{g}{L\,s} \cdot \left[L\,(s - u) - \frac{u}{g}\,v^2\right] \qquad (1)$$

The initial acceleration is obtained for v = 0 and is:

$$A_o = (\frac{dv}{dt})_o = g\,(1 - \frac{u}{s}) \qquad (2)$$

A_o depends only on the specific gravity of the particle and not on its dimensions. The initial accelerated motion thus permits efficient separation, but it is soon followed by an almost uniform motion with a velocity close to the terminal velocity (V_t) which is obtained for $\frac{dv}{dt} = 0$ and is:

$$V_t = \sqrt{g\,L\,(\frac{s - u}{u})} \qquad (3)$$

Therefore, (V_t) depends on both the specific gravity and the dimensions of the particle. The optimum conditions for gravity separation are thus obtained during the initial accelerated fall. These conditions are repeated as frequently as possible in jigs by pulsating motions of the medium.

Consequently, it is very important to calculate the size and duration of the initial accelerated motion.

ANALYSIS OF THE MOTION

Duration of the Accelerated Motion

The equation of motion (1) can be written:

$$\frac{dv}{dt} = g\left(\frac{s-u}{s}\right)\left[1 - v^2\frac{u}{g\,L\,(s-u)}\right]$$

and becomes by substitution of equations (2) and (3):

$$\frac{dv}{dt} = A_o(1 - v^2/V_t^2) \tag{4}$$

$$t = \frac{1}{A_o}\int\frac{dv}{1 - v^2/V_t^2}$$

$$t = \frac{1}{A_o}\,(-V_t/2)\,\mathrm{Log}_e\left(\frac{1 - v/V_t}{1 + v/V_t}\right) + C$$

If $t = 0$, we have $v = 0$, and therefore the integration constant $C = 0$

$$t = -\frac{V_t}{2A_o}\,\mathrm{Log}_e\left(\frac{1 - v/V_t}{1 + v/V_t}\right) \tag{5}$$

The duration (T) of the accelerated motion is infinite since it is obtained for $v = V_t$. However, an approximate value of (T) can be obtained when the velocity of the particle is v_1 equal to 90% of the terminal velocity V_t. Then the acceleration is:

$$A_1 = \left(\frac{dv}{dt}\right)1 = A_o\,(1 - v_1^2/V_t^2)$$

As $v_1 = 0.9\,V_t$, we have $A_1 = 0.19\,A_o$

If this acceleration A_1 is not regarded as negligible, another percentage of V_t can be used for the calculation such as $v_2 = 0.95\,V_t$.

The duration (t) of the accelerated motion of the particle up to the time when its velocity is 90% of the terminal velocity will be obtained by replacing (v) by (v_1) in equation (5). If desirable, some other percentage of the terminal velocity may be used.

Distance of fall in Accelerated Motion

From equation (4): $\frac{dv}{dt} = A_o(1 - v^2/V_t^2)$ (4)

we have: $A_o\, dt = dv/(1 - v^2/V_t^2)$

Hence:

$$v = V_t(1 - e^{-2(A_o/V_t)t}) / (1 + e^{-2(A_o/V_t)t}) + C \qquad (6)$$

If $t = 0$, we have $v = 0$ and therefore the integration constant $C = 0$. Then, if (h) is the distance of fall of the particle, we have: $dh/dt = v$ and by integration:

$$h/V_t = \int dt - 2\int \frac{e^{-2(A_o/V_t)t}}{1 + e^{-2(A_o/V_t)t}}\, dt$$

$$= t + (V_t/A_o)\, \mathrm{Log}_e\, (1 + e^{-2(A_o/V_t)t}) + C$$

If $t = 0$, we have $h = 0$ and the integration constant $C = -(V_t/A_o)\, \mathrm{Log}_e\, 2$. Hence:

$$h = V_t\, t + (V_t^2/A_o)\, \mathrm{Log}_e\, \left(\frac{1 + e^{-2(A_o/V_t)t}}{2}\right) \qquad (7)$$

From equations (2) and (3), we have: $V_t^2/A_o = Ls/u$

From equation (5), we have:

$$e^{-2(A_o/V_t)t} = (1 - v/V_t) / (1 + v/V_t)$$

By substituting this value in (7), and also replacing (t) by formula (5) and V_t by formula (3), we obtain:

$$h = -\frac{Ls}{2u}\left[\mathrm{Log}_e\, \left(\frac{1 - v/V_t}{1 + v/V_t}\right) + 2\, \mathrm{Log}_e\, (1 + v/V_t)\right]$$

$$h = -\frac{Ls}{2u}\, \mathrm{Log}_e\, (1 - v^2/V_t^2) \qquad (8)$$

If the expressions (6) and (7) of the velocity (v) and the distance (h) in function of time are not desired, the relation (8) can be obtained much more quickly as follows:

From equation (4) we have:

$$v = \frac{dh}{dt} = \frac{dh}{dv} \cdot \frac{dv}{dt} = \frac{dh}{dv} A_o (1 - v^2/V_t^2)$$

Hence: $$h = \int dh = \frac{1}{A_o} \int \frac{v dv}{1 - v^2/V_t^2} = \frac{-V_t^2}{2A_o} \text{Log}_e (1 - v^2/V_t^2) + C$$

If $t = 0$, we must have $h = 0$ and therefore the integration constant $C = 0$.

From equations (2) and (3), we have:

$$V_t^2/A_o = Ls/u$$

and thus this expression of (h) is the same as equation (8).

The distance of fall (H) of the particle in accelerated motion is infinite since it is obtained for $v = V_t$.

However, here again, an approximate value of the fall obtained while the acceleration is reasonably efficient can be obtained up to the point where the velocity of the particle is $v_1 = 90\%$ of the terminal velocity V_t, or a higher percentage if desired.

EXAMPLES

(1) $L = 10$ cm, $s = 2$, $v_1 = .9\ V_t$

With $u = 1$, we find: $t = 3/10$ second and $h = 16.6$ cm.

With $u = 1.5$, we find: $t = 3.5/10$ second and $h = 11.1$ cm.

(2) $L = 1$ mm, $s = 2$, $v_1 = .9\ V_t$

With $u = 1$, we find: $t = 3/100$ second and $h = 1.7$ mm.

With $u = 1.5$, we find: $t = 3.5/100$ second and $h = 1.1$ mm.

CONCLUSIONS

Even though the assumptions, on which this mathematical analysis is based, may not be perfectly correct, the results obtained from the calculated formulas for the terminal velocity and the duration and distance of fall in accelerated motion can be favourably compared with experimental data, and they definitely show that the motion completed with notable acceleration is of very short duration and extremely limited amplitude for very small particles. Therefore, unless the frequency of pulsations is rather high, each particle under consideration will be most of the time submitted to a motion with uniform velocity approximately equal to the asymptotic terminal velocity, instead of the initial accelerated motion which corresponds to the optimum conditions for separation according to gravity.

Instead of repeating more frequently these optimum conditions by increasing the frequency of pulsations, which may be limited for mechanical reasons, these conditions may be prolonged by extending the suction phase over a longer period, thereby maintaining within a low range the relative velocity of the particles in the accelerating descending medium. Thus, the initial high acceleration motion can be obtained during a longer part of the jigging cycle. Attempts to produce a somewhat longer suction phase have been quite successful in many cases of fine coal jigging, but it seems that better gravity concentration could be possible in jigs washing fine coal, by artificially prolonging the duration of the suction phase and at the same time increasing the frequency of pulsations by reducing to the utmost the duration of the upward stroke or pulsion phase.

This may be facilitated by using a battery of small well controlled jigs instead of one large jig, containing a great mass of water in rather uncontrolled oscillation.

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